Practical multivariable control for multi-axis hydraulic servosystems

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Abstract

A practical and effective approach to controller design for a multi-axis servosystem is to transform measured feedback variables into an alternative co-ordinate system which reduces cross-axis coupling. A general transformation framework is presented in this paper for parallel-actuator systems, including those with overconstraint (i.e. with more actuators than rigid body degrees-of-freedom). In this case force control for the extra axes is recommended, and appropriate transformations from measured actuator positions and forces to controlled position and force are given.

Keywords

Multi-axis control, electrohydraulic servosystems, modal control, motion control, redundant actuation, overconstraint

1 Introduction

Many industries require machines which can control motion or force simultaneously in several linear or rotary axes. The machine tools industry is one example. The testing and simulation industry is another, where in-service motion or force usually needs to be replicated; this ranges from automotive durability testing, to earthquake simulation, and motion-based flight training simulators. Accurate multi-axis control of motion and/or force is challenging due to coupling between different axes and complex dynamic characteristics which cannot easily be corrected by simple closed-loop controllers.

A number of practical techniques have been developed and are in industrial usage which tackle specific problems in multi-axis control. In the testing industry these have been shown to be very effective. They include compensation for interaction due to geometric effects, such as co-ordinate transformation and 'valve cross-compensation' [1] The purpose of this paper is not to propose a new control method, but instead to show that a number of practical techniques currently in use can be treated as examples of one general co-ordinate transformation method. This allows additional insight to the control of new multi-axis machines.

To focus the work, the main field considered is machines for the dynamic testing of components and structures. These machines have certain characteristics:

- motion ranges are often small compared to the machine size, and it is often acceptably accurate to linearise kinematic relationships,
- parallel rather than serial mechanisms are used,
- there is sometimes overconstraint, i.e. more actuators than degrees of freedom (DOF),
- actuation is usually by servo-hydraulics, and the dynamic response of each actuator is similar,
- displacement and force are normally measured in actuator co-ordinates.

Examples of this type of machine are shaker tables for earthquake simulation, and automotive suspension test rigs. In this paper, once the general control approach has been formulated, its application to some specific example test rigs is shown.

The key element in the approach is to transform the co-ordinate system from actuator space to specimen (or end-effector) space, and implement closed-loop control on the transformed motion variables. There are choices as to how the specimen motion is defined, including definition in terms of individual actuator displacements (a one-to-one transformation), so the framework does not exclude individual actuator control. However, in many cases it has been found that using true specimen related co-ordinates can give accurate dynamic decoupling, for example modal control of shaking tables [2,3] or more simply using Cartesian co-ordinates can give approximate decoupling [1]. Such benefits are not explored further here.

Many structural test rigs are overconstrained. There are two reasons for this:

1. the purpose of some systems is to exert forces on a specimen directly as well as to impart particular test motions,
2. multiple smaller actuators are used instead of one large actuator to spread the load and simplify the mechanical design.

In practice, it is essential to control the extra actuation variables in force rather than position loops. Otherwise errors in kinematic relationships or position measurement tend to lead to actuators 'fighting' one another, and the potential destruction of the machine. Some previous research into redundant parallel mechanisms has been published for robotics applications [4,5], but the necessity to incorporate internal force control has only sometimes been recognised [6]. Over actuated rigs also provide more options for how motion in specimen co-ordinates is calculated, which in turn limits the choice of a consistent set of force variables. The consistency between motion and force variables is important to avoid coupling from displacement to force-control axes.

In this paper, the general co-ordinate transformation framework is developed first, and then two industrial case studies are described, both multi-axis servohydraulic test rigs.

2 Co-ordinate transformation framework

2.1 Position transformations

Figure 1 shows a generic test specimen with m rigid body degrees of freedom, where

$$0 \leq m \leq 6 \tag{1}$$

Forces are exerted on the specimen by n actuators, where

$$n \geq m \tag{2}$$

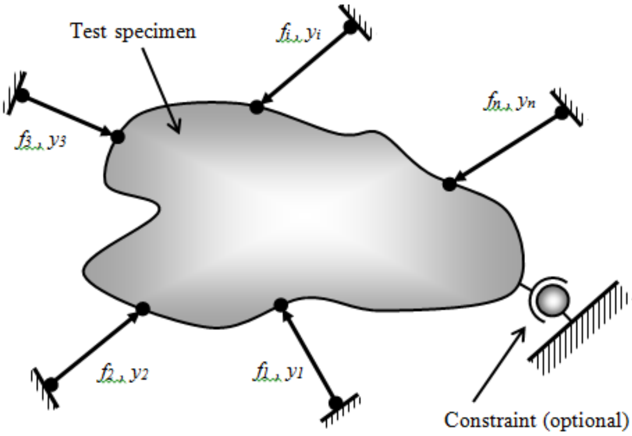

Fig. 1 *A generic test system*

The displacement of the actuators from a defined zero position, and the direction in which the forces act, are uniquely determined by the combination of the gross movement of the specimen, together with the specimen's structural deformation. It will however be assumed that motions are small so that the force directions can be approximated as constant.

The gross displacement of the specimen is represented by a vector $\boldsymbol{y_c}$ whose elements are linear or angular displacements; these are the rigid body displacements which the user wishes to control, and their definition depends on the application. The remaining displacement axes relate to the deformation of the specimen, represented by vector $\boldsymbol{y_d}$; it is usually the forces associated with these axes which need to be controlled. For small motions, a linear transformation approximates the relationship between the specimen displacement (in both rigid body and deformation co-ordinates) to actuator displacement:

$$\begin{bmatrix} \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{y_c} \\ \boldsymbol{y_d} \end{bmatrix} = \boldsymbol{y} \tag{3}$$

where $$\boldsymbol{y_c} = \begin{bmatrix} y_{c,1} & y_{c,2} & \cdots & y_{c,m} \end{bmatrix}^T \tag{4}$$

$$\boldsymbol{y_d} = \begin{bmatrix} y_{d,1} & y_{d,2} & \cdots & y_{d,n-m} \end{bmatrix}^T \tag{5}$$

$$\boldsymbol{y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_n \end{bmatrix}^T \tag{6}$$

and **C** and **D** are rectangular matrices, $\mathbf{C} \in M_{n,m}$ and $\mathbf{D} \in M_{n,n-m}$.

C is dictated by the definition of $\boldsymbol{y_c}$. Two issues which need to be addressed are how to calculate $\boldsymbol{y_c}$ from the actuator displacements $\boldsymbol{y}$, and how to define the deformation co-ordinates (i.e. how to choose **D**).

Let $\mathbf{P} \in M_{m,n}$ be chosen to transform from actuator to specimen rigid body co-ordinates:

$$\boldsymbol{y_c} = \mathbf{P}\boldsymbol{y} \tag{7}$$

This always holds, including when the deformation displacements are zero, in which case from Eq (3):

$$\mathbf{C}\boldsymbol{y_c} = \boldsymbol{y} \qquad (\text{for } \boldsymbol{y_d} = 0) \tag{8}$$

Comparing Eqs (7) and (8) gives the requirement that:

$$\mathbf{PC} = \mathbf{I_m} \tag{9}$$

where $\mathbf{I_m} \in M_{m,m}$ is an identity matrix.

The choice of **P** is not unique, and may depend on the requirements for a particular application. One solution is to choose **P** as the pseudo inverse of **C**:

$$\mathbf{P} = \left(\mathbf{C^T C}\right)^{-1}\mathbf{C^T} \tag{10}$$

This will be referred to as the least squares solution, as it gives the best fit of the rigid body related displacements to the actual actuator displacements in a least squares sense.

To determine **D**, consider Eq (3) pre-multiplied through by P:

$$\mathbf{PC}\boldsymbol{y_c} + \mathbf{PD}\boldsymbol{y_d} = \mathbf{P}\boldsymbol{y} \tag{11}$$

Substituting in Eqs (7) and (9), and noting that in general $\boldsymbol{y_d} \neq 0$, this gives:

$$\mathbf{PD} = 0 \tag{12}$$

Thus the columns of **D** are the vectors which form the basis of the null space of matrix **P**. Any solution for **D** is not unique; if $\mathbf{D_1}$ is one solution, then other solutions are given by:

$$\mathbf{D} = \mathbf{D_1 K} \tag{13}$$

where $\mathbf{K} \in M_{n-m,n-m}$ is full rank. The solution used in this paper is found by a simple rearrangement of the row-reduced echelon form (RREF) of **P** [7], implemented by the Matlab® *null(P,'r')* command.

Note that using the least squares solution for **P**, Eq (12) reduces further to $\mathbf{C^T D} = 0$.

2.2 Force transformations

In principle, the displacement of the deformation axes could be controlled, in which case a way of calculating the deformation displacements from the actuator displacements is required. Defining this transformation matrix as $\mathbf{R} \in M_{m-n,n}$:

$$\boldsymbol{y_d} = \mathbf{R}\boldsymbol{y} \tag{14}$$

However, in almost all testing industry applications, the deformation forces are controlled (or occasionally local strains). Thus the derivation here is for the definition and calculation of the deformation-related forces. The work done by the actuators is the sum of the work done in rigid body and deformation co-ordinates, i.e.

$$\boldsymbol{f_c}^T \boldsymbol{y_c} + \boldsymbol{f_d}^T \boldsymbol{y_d} = \boldsymbol{f}^T \boldsymbol{y} \tag{15}$$

where the force vectors are

$$\boldsymbol{f_c} = \begin{bmatrix} f_{c,1} & f_{c,2} & \cdots & f_{c,m} \end{bmatrix}^T \tag{16}$$

$$\boldsymbol{f_d} = \begin{bmatrix} f_{d,1} & f_{d,2} & \cdots & f_{d,n-m} \end{bmatrix}^T \tag{17}$$

$$\boldsymbol{f} = \begin{bmatrix} f_1 & f_2 & \cdots & f_n \end{bmatrix}^T \tag{18}$$

Substituting for $\boldsymbol{y_c}$ using Eq (7) and for $\boldsymbol{y_d}$ using Eq (14) gives:

$$\boldsymbol{f_c}^T \mathbf{P}\boldsymbol{y} + \boldsymbol{f_d}^T \mathbf{R}\boldsymbol{y} = \boldsymbol{f}^T \boldsymbol{y} \tag{19}$$

This equality is true for all $\boldsymbol{y}$, and so reduces to:

$$\boldsymbol{f_c}^T \mathbf{P} + \boldsymbol{f_d}^T \mathbf{R} = \boldsymbol{f}^T \tag{20}$$

or

$$\mathbf{P^T}\boldsymbol{f_c} + \mathbf{R^T}\boldsymbol{f_d} = \boldsymbol{f} \tag{21}$$

Define $\mathbf{Q} \in M_{m-n,n}$ as the transformation from actuator to deformation forces:

$$\boldsymbol{f_d} = \mathbf{Q}\boldsymbol{f} \tag{22}$$

This equation always holds, including when the rigid body forces are zero, in which case from Eq (21):

$$\mathbf{R^T}\boldsymbol{f_d} = \boldsymbol{f} \qquad (\text{for } \boldsymbol{f_c} = 0) \tag{23}$$

Comparing Eqs (22) and (23) gives the requirement that:

$$\mathbf{QR^T} = \mathbf{I_{n-m}} \tag{24}$$

Pre-multiplying Eq (21) by **Q**:

$$\mathbf{QP^T}\boldsymbol{f_c} + \mathbf{QR^T}\boldsymbol{f_d} = \mathbf{Q}\boldsymbol{f} \tag{25}$$

Thus substituting Eqs (22) and (24) into (25), and noting that in general $f_c \neq 0$, shows that:

$$\mathbf{QP^T} = 0, \quad \text{or} \quad \mathbf{PQ^T} = 0 \tag{26}$$

By comparison with Eq (12), **Q** can be the transpose of **D**:

$$\mathbf{Q} = \mathbf{D^T} \tag{27}$$

This solution is not unique, but it will be used throughout this paper.

2.3 Controller

The closed-loop controller concept is shown in Figure 2. Loop closure is done in specimen co-ordinates. The rigid body axes are position controlled, and the deformation axes are force controlled. The control signals in vector $\boldsymbol{u}$ are assumed to correspond to actuator velocity commands. This is a reasonable assumption for hydraulic actuation [1]. Hence differentiating Eq (3) with respect to time gives a velocity transformation which can be applied to the control signal vector:

$$[\mathbf{C} \quad \mathbf{D}]\begin{bmatrix} \boldsymbol{u}_c \\ \boldsymbol{u}_d \end{bmatrix} = \boldsymbol{u} \tag{28}$$

As will be shown by example, control in specimen co-ordinates provides the potential for decoupling and simplification of the individual loop dynamics.

The process for determining the transformation matrices can be summarised as follows.

1. Select **C** (depends on the requirements of application).
2. Select a **P** such that $\mathbf{PC} = \mathbf{I_m}$ (Eq (9)). One option for is the least squares solution, $\mathbf{P} = \left(\mathbf{C^T C}\right)^{-1}\mathbf{C^T}$ (Eq (10)).
3. Determine **D** from $\mathbf{PD} = 0$ (Eq (12)), by rearranging the RREF of **P**. Note that multiplying **D** by a scalar might be desirable so that the transformed forces are physically meaningful.
4. Determine **Q** from $\mathbf{Q} = \mathbf{D^T}$ (Eq (27)).

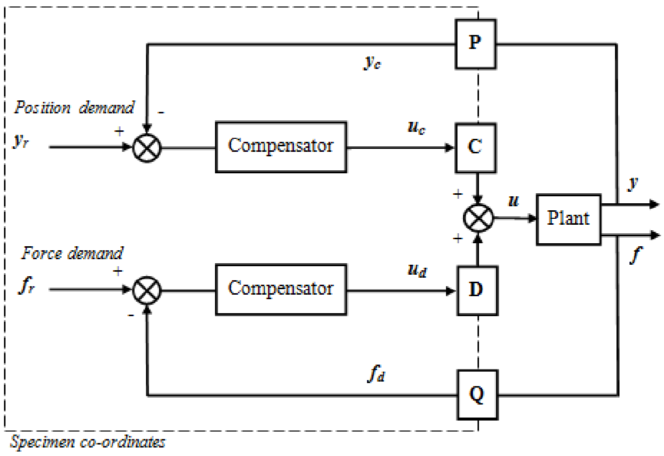

Fig. 2 *Multi-axis control with co-ordinate transformations*

3. Industrial case studies

3.1 Cruciform materials testing machine

In this system (Fig. 3), the requirement is to load the specimen in two orthogonal directions simultaneously, while the centre of the specimen remains fixed to avoid side-loading the actuators. Such testing machines have been built in small numbers for several years. Considering just one pair of antagonistic actuators, $m = 1$ and $n = 2$.

To control the specimen centre position it must be defined as the displacement variable y_c (the demand will

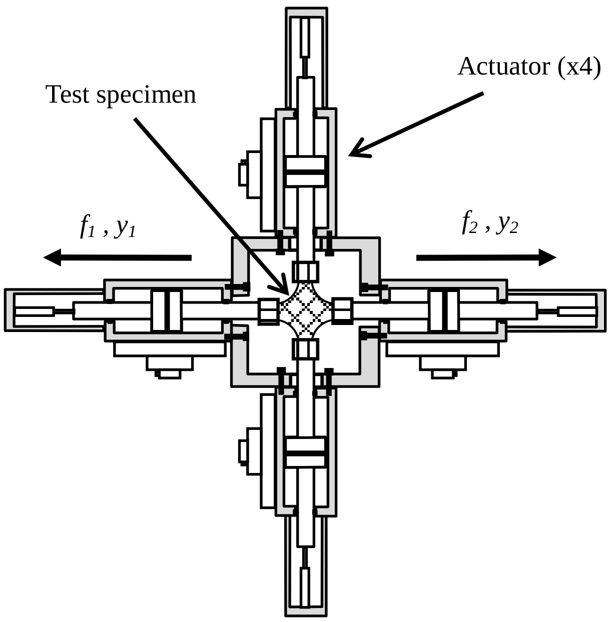

Fig. 3. *Cruciform testing machine (force directions refer to loads acting on the specimen)*

always be zero). This gives $\mathbf{C}=[-1\quad 1]^T$. Using the least squares solution, $\mathbf{P}=[-0.5\quad 0.5]$, giving $\mathbf{D}=[0.5\quad 0.5]^T$ and $\mathbf{Q}=[0.5\quad 0.5]$. The development of an industrial controller for this type of machine, which derives this same set of transformations heuristically, is described in [8]

3.2 Shaking table with 8 actuators

Large shaking tables are used for earthquake simulation, and they often have more actuators than degrees of freedom to spread the loads more evenly on the table structure. Fig. 4 shows a 6 DOF shaking table with 3mx3m surface at the University of Bristol in the UK [9]. It has 8 actuators (m = 6 and n = 8).

Defining the controlled displacements as x, y, and z translations, and roll, pitch, and yaw rotations respectively, and taking actuator retraction as positive, the actuator spacings in this case give (in metres):

$$\mathbf{C}=\begin{bmatrix} 0 & 0 & -1 & -1.25 & 1.25 & 0 \\ 0 & 0 & -1 & 1.25 & 1.25 & 0 \\ 0 & 0 & -1 & -1.25 & -1.25 & 0 \\ 0 & 0 & -1 & 1.25 & -1.25 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1.8 \\ 0 & 1 & 0 & 0 & 0 & -1.8 \\ 1 & 0 & 0 & 0 & 0 & -1.8 \\ -1 & 0 & 0 & 0 & 0 & -1.8 \end{bmatrix}$$

Using the least squares solution:

$$\mathbf{P}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & 0.5 & 0 & 0 \\ -0.25 & -0.25 & -0.25 & -0.25 & 0 & 0 & 0 & 0 \\ -0.2 & 0.2 & -0.2 & 0.2 & 0 & 0 & 0 & 0 \\ 0.2 & 0.2 & -0.2 & -0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.14 & -0.14 & -0.14 & -0.14 \end{bmatrix}$$

which gives $\mathbf{D}^T=\begin{bmatrix} 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \end{bmatrix}$

Thus the deformation axes relate to twist and shear of the table. Control using these co-ordinate transformations is currently operational for this table.

5. Conclusions and discussion

The transformation framework presented here can be applied to a number of parallel-actuator motion system scenarios:

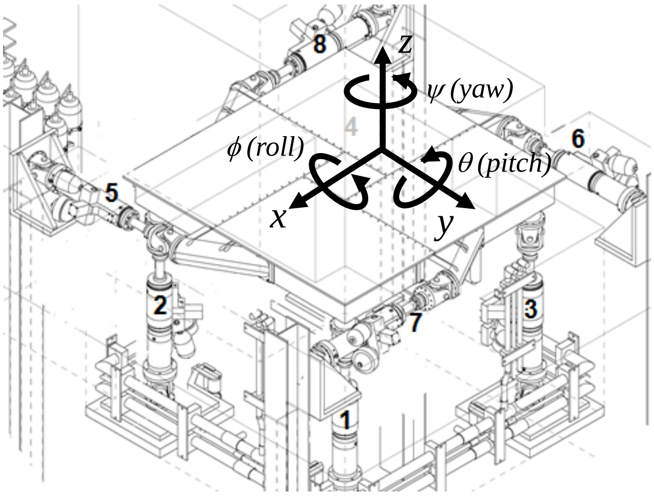

Fig. 4 *Six degree-of-freedom shaker table with 8 actuators. Origin position is centre of table in horizontal actuator plane.*

1. Square systems (no overconstraint). In this case the position transformations follow simply from the user-defined co-ordinates defining the rigid body motion of the specimen (or other type of end-effector). In the testing industry, co-ordinate systems which achieve decomposition of the resonant modes have been tried, calculated from stiffness and mass matrices (Plummer, 2007b). However a much more common approach is to use Cartesian co-ordinate control variables, with an origin near the centre of the test specimen, as this often provides approximate modal decomposition without the need to determine stiffness or mass parameters. The advantages of modal or approximate modal control are not explored in this paper.
2. Overconstrained systems. Additional actuators are sometimes needed to generate internal stresses in a specimen (in which case there is overconstraint but not redundancy), or to simplify the structural design of the test rig. Force control of additional transformed axes is desirable, and the definition of these axes is restricted by the need to decouple them from the position axes. The solution for the force axis transformation matrix given this restriction is the basis of the null space of a matrix and thus not unique, but examples use a particular method for calculating this basis. In practice, in the testing industry, appropriate co-ordinate transformation is often applied to geometrically-simple multi-axis

systems, but the new framework allows more complex systems to be tackled.

3. Cross-compensation in simple multi-axis systems. For simple overconstrained systems (typically 2 axes), adding-in scaled control signals from other axes is a practical modification to individual actuator controllers to overcome coupling from position to force axes. Such an approach can also be derived using the proposed framework.

The contribution of this paper is to present existing practical co-ordinate transformations in a unified framework, and provide a basis for new applications. Due to the limited range of motion of many testing systems, using linear approximations for kinematic transformation is acceptable. While this may also be true for some other applications, many will require non-linear transformations for sufficient accuracy.

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